PCM (Pulse Code Modulation) is a standardized method used in the telephone network (POTS) to change an analog signal to a digital one. The analog signal is first **sampled** at a 8-kHz sampling rate. Then each sample is **quantized** into 1 of 256 levels and then **encoded** into digital eight-bit words. This process is illustrated in the Figure below.

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**The three steps for PCM**

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Sampling

After sampling, the signal value is known only at discrete points in time, called sampling instants. If these points have a sufficiently close spacing, a smooth curve drawn through them allows us to interpolate intermediate values to any degree of accuracy (see Shannon’s Theorem). We can therefore say that a continuous curve can be adequately (i.e. perfectly, in theory) described by the sample values alone.

If the sampling frequency, $fs$, is higher than two times the highest frequency component of the analog signal, $B$, the original analog signal is completely described by these instantaneous samples alone. That is, $fs > 2B$. This minimum sampling frequency is sometimes called the Nyquist rate. The sampling time $Ts$ is:

$$Ts = 1/ fs < 1/(2B)$$

and $1/(2B)$ is the maximum sampling time.
Recovery of signal after sampling

The frequency-domain description show the spectrum of the analog signal, \( x(t) \), say \( X(f) \), and that of the sampled signal \( y(t) \), say \( Y(f) \). Before sampling, \( X(f) \) contains speech frequencies up to 3.4 kHz.

As an example of the frequency components of speech in figure below see the 1-kHz cosine wave as a solid spectral line at the 1-kHz point on the frequency axis.
After sampling, we have components at 1 kHz, 8 kHz – 1 kHz = 7 kHz, and at 8 kHz + 1 kHz = 9 kHz, as seen in the figure. In addition to these components, sampling also generates components around double sampling frequency, three times sampling frequency, and so on.

The reproduction of an original signal from a sampled signal is performed by a lowpass filter and in the case of voice the bandwidth \( B = 4 \text{ kHz} \), that is, half the sampling frequency.

Warning! If \( f_s < 2B \) the message spectra around zero frequency and following spectral component will overlap! This phenomenon is called “aliasing”. Under aliasing condition the analog signal \( x(t) \) cannot be reconstructed.
Quantization

To transmit the sampled values via a digital system, we have to represent each sample value in numerical form. This requires **quantizing** where each accurate sample value is rounded off to the closest numerical value in a given numerical set.

In the quantizing process the information in accurate signal values **is lost because of rounding off** and the original signal cannot be reproduced exactly any more.

The more quantum levels we use, the better performance we get.

For **binary coding**, the number of quantum levels is

\[ q = 2^n \]

where \( q \) denotes the number of quantum levels and \( n \) is the length in bits of the binary codewords that describe the sample values.
Bandwidth and quantization noise

Examples:
  a) voice signal (Telephony): \( q = 256 \) (quantized signal levels); \( n = 8 \) (n. of bit per codeword)
  b) music signal (CD recording): \( q = 65,536 \) (quantized signal levels); \( n = 16 \) (codeword).

The better quality we require, the more quantization levels we need and the longer codewords we need. This leads to the requirement of a higher bit rate for transmission of the data representing the original message (in real-time) and, finally, larger bandwidth.

Quantum noise power is the variance of quantizing error:

\[
N = \sigma_q^2 = \frac{1}{3q^2}
\]

Signal to quantization noise ratio:

\[
SQR = \frac{S}{N} \leq 3q^2
\]
Nonuniform Quantizing

Linear quantizing is not the optimum solution because at low signal levels the quantizing noise is high and the S/N is very low. At high signal levels the quantizing noise is the same even though we would tolerate a high noise level. We should define quantizing levels in such a way that performance is acceptable over a wide dynamic range of the voice.

In nonuniform quantizing we use more code words and we have a shorter distance between quantum levels for low-level samples and allow higher quantizing distortion at high-level samples.

Expanding/Compressing process known as **companding**
Characterstic curve of the compressor

Compresser characteristic

Expandor: $x = Z^{-1}(y)$
Companding laws

One family of compression characteristics (Recommendation G.733) used in North America and Japan is the **μ-law companding**, which is defined as follows:

\[
Z(x) = \text{sgn}(x) \cdot \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)}
\]

**A-law companding** (Recommendation G.732) is used as the European standard, where the curve is divided into linear and logarithmic sections:

\[
Z(x) = \begin{cases} 
\text{sgn}(x) \cdot \frac{1 + \ln A|x|}{1 + \ln A} & \text{for } \frac{1}{A} < |x| < 1 \\
\frac{Ax}{1 + \ln A} & \text{for } -\frac{1}{A} < x < \frac{1}{A}
\end{cases}
\]

For A-law companding, the SQR is **constant** in the logarithmic section and **directly proportional** to the signal value in the linear section.
Binary Coding

In PCM encoding process each sample is represented as one in the set of **eight-bit binary words**. As an example of binary coding, the structure of the eight-bit binary word in the case of European PCM coding, **the A-law, is defined in the following way**:

**Bit 1**, the most significant bit (MSB): is the first bit, reveals the polarity of the sample; “1” represents positive polarity, “0” is negative polarity.

**Bits 2, 3, and 4**: define the segment where the sample value is located. Segments “000” and “001” together form a linear curve for low-level positive or negative samples. An A-law curve has 13 linear sections as shown in Figure.

**Bits 5, 6, 7, and 8**: least significant bits (LSBs), they reveal the quantized value of the sample inside one of the segments. Thus each segment is divided in a linear fashion into 16 values (quantum levels).
PCM encoder and decoder

PCM encoder

PCM decoder
In DPCM only the difference between a sample and the previous value is encoded as shown in Figure.

Because the difference is typically much smaller than the overall value of the sample, we need fewer bits for the same accuracy as in ordinary PCM and the required bit rate is reduced:

- 5 bits (polarity and 4 bits for 16 quantum levels)
Further compression is achieved by adapting the predictor and the quantizer to the characteristics of the signal. Both the encoder and the decoder use the same algorithm to estimate the values of the following samples with help of the preceding samples, and only the error to this estimate is transmitted as in DPCM.

To further reduce the number of bits per sample, ADPCM adapts quantizing levels to the characteristics of the analog signal.

In the original 32-Kbps ADPCM method, the difference between the predicted and actual sample value is coded with four bits, that is, into 15 quantum levels, and the data rate is half that of conventional PCM.
More complex coding schemes: GSM

The principle of GSM speech coding