Nonlinear viscoelastic bodies under live loads

Stuart S. Antman[1], Alexey B. Stepanov[2], Süleyman Ulusoy[3]

[1]University of Maryland, College Park, USA

[2] University of Illinois at Urbana-Champaign, USA

[3] Zirve University, Gaziantep, Turkey

ABSTRACT

This lecture describes new results for steady states and motions of nonlinearly (visco)elastic bodies subject to the live loads of hydrostatic pressure, centrifugal force, and slip-stick friction. The problems treated are those for cylindrical and spherical shells undergoing radial motions and for the motion in space of rods carrying rigid bodies. The analyses depend critically on new, refined constitutive restrictions for these materials. In particular, for equilibrium problems, this lecture treats the role of the constitutive equations in existence and nonexistence, uniqueness and nonuniqueness, and qualitative behavior. For dynamical problems, it treats their role in preclusion of total compression, existence global in time, infinite-time blowup, finite-time blowup, and the presence of attractors.

Mathematical modeling of the elasto-hydrodynamics of lipid membranes

Marino Arroyo

Universitat Politécnica de Catalunya, Barcelona, Spain

ABSTRACT

Lipid membranes are fundamental separation structures in animal cells, which are able to deal with conflicting requirements: they provide structural integrity and shape to multiple cell organelles and at the same time they are very malleable and constantly remodel. They accomplish this thanks to their in-plane fluidity and out-of-plane elasticity. Furthermore, their bilayer architecture is crucial to understand many important phenomena involving monolayer asymmetry. I will describe continuum models for lipid bilayers describing their elasto-hydrodynamics, and will apply these models to understand the dynamics of lipid membranes in specific situations.

Mathematical problems related to the mechanical stability of viral capsids and protein complexes

Paolo Cermelli

University of Turin, Italy

ABSTRACT

Viral capsids are interesting mechanical objects in that they are highly stable assemblies of proteins kept together by relatively weak bonds. In this work we use capsids as case studies to investigate the physics of such assemblies, by focusing on the disassembly process occurring when the capsid opens to release the genome inside the cell. Under very general assumptions on the interaction potential between the capsomers, we prove that it is energetically favorable that destabilization occurs as a cascade of detaching events propagating along the capsid, triggered by a single event at an isolated bond. Then, we enrich the physics by taking into account stochastic variations of the bonds that drive the system beyond the destablization threshold. Using the large deviations principle we show that again the opening of the capsid occurs with high probability by a cascade of destabilization events propagating along the shell. Possible generalizations to other complex interacting systems are discussed. This work is in collaboration with G. Indelicato (Torino, Italy) and R. Twarock (York, UK).

Balance of Linear Momentum, Newton's Third Law and Generalized Continua

Giampietro Del Piero

Dipartimento di Ingegneria, Università di Ferrara, Ferrara, Italy.

International Research Center M&MoCS, Cisterna di Latina, Italy

ABSTRACT

The papers [1] by Gurtin and Martins and [2, 3] by Šilhavy brought new light on the foundations of continuum mechanics. Their main result is that the existence of the Cauchy stress is a consequence of certain regularity properties of the system of the contact actions, and not of the balance law of linear momentum, as commonly believed before. This solves a basic problem in the mechanics of generalized continua. Indeed, in a generalized continuum each independent variable requires its own balance law. By analogy with the classical continua, there is a tendency to attributing to each balance law the status of a general law of mechanics, and this leads to an embarrassing proliferation of general laws.

Deducing the stress measures from regularity assumptions eliminates this inconvenience. With the stress measures at disposal from the very beginning, the balance laws, now downgraded to *pseudobalance* laws, can be deduced from a single general principle, the indifference of power, as shown by Noll in the special case of the classical continuum [4].

The required regularity includes the assumption that the contact actions acting on surfaces which are boundaries of a body, are additive on disjoint bodies. As stated by Noll in [5], this is possible only if the contact actions are defined on oriented surfaces, and are skew-symmetric with respect to change of orientation. This property can be identified with the *law of action and reaction*, or Newton's third law. So this law, which in classical continuum mechanics is a mere corollary of the balance of linear momentum, in the broader context of generalized continua recovers its original status of a general law.

In the proposed communication, which reflects the contents of the papers [6, 7], I will show how much this new approach simplifies the structure of the mechanics of generalized continua.

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Composite thin-walled beams by Γ-convergence: from theory to applications

Cesare Davini

University of Udine, Italy

ABSTRACT

The behavior of thin-walled beams does not fit the De Saint-Venants theory of beams and a multitude of *ad hoc* models have been proposed throughout the years, starting from that of Vlasov. In two joint papers with R. Paroni and L. Freddi we have considered a beam whose cross-section is a tubular neighborhood of some simple curve, γ, for the two instances where the curve is either open or closed. We assumed that the wall thickness scales with a parameter δ_ε and the length of γ with ε, where δ_ε goes to zero faster than . Starting from three dimensional linear elasticity, we derived the Γ-limit problem for the case in which the ratio between ε^2 and δ_ε remains bounded. Depending upon the limit value of that ratio, two one-dimensional asymptotic models are so obtained; the deduction applies to a fully anisotropic and inhomogeneous material, thus making the theory suitable for composite thin-walled beams. The approach recovers in a systematic way many features of the beam model in Vlasovs theory. Here I focus on establishing a bridge between those mathematical results and their implementation to a real problem.

Variational methods and functionals defined on BD

Marco Degiovanni

Catholic University of the Sacred Hearth, Brescia, Italy

ABSTRACT

When variational methods, which have greatly developed in the setting of nonlinear elliptic equations, are applied to problems coming from Continuum Mechanics, the standard assumptions are usually not satisfied. We aim to show some typical difficulties and propose an extended approach which is suitable, for instance, in the setting of functionals defined on BD.

Kinematics and energetics of unstretchable two-dimensional elastic bodies

Eliot Fried

Okinawa Institute of Technology, Japan

ABSTRACT

We will present a variational theory for two-dimensional bodies that are unstretchable in the sense that they are capable of sustaining only isometric deformations. Aside from the relevant Euler–Lagrange equations, we will derive and discuss boundary conditions, including those needed to describe Möbius bands. An elementary application of the theory will also be considered.

A Geometric Theory of Nonlinear Morphoelastic Shells

Alain Goriely

Oxford University, UK

ABSTRACT

Various theories of shells have been used to model growing biological membranes or membranes with defects. Physicists typically favour Helfrich-type models that assume that the membrane is fluid along its surface but resists bending. Mechanicians typically use incompatible nonlinear shell theories. Here we revisit these different theories and formulate a general geometric theory of nonlinear morphoelastic shells that can model either the time evolution of residual stresses induced by bulk growth or distributed defects in two-dimensional bodies. In this geometric theory, growth or defects are modeled using an evolving referential configuration for the shell. We consider the evolution of both the first and second fundamental forms in the material manifold by considering them as dynamical variables in the variational problem. Their evolution can be used to model both surface growth, defects, and remodeling (the evolution of spontaneous curvatures). Then, we use a Lagrangian field theory to derive the governing equations of motion. We also consider the particular case where defects can be modeled by a Rayleigh potential. This talk is based on joint work with Souhayl Sadik, Arzhang Angoshtari, and Arash Yavari.

The energy of a Möbius band

Roberto Paroni

University of Sassari, Italy

ABSTRACT

In 1929 Sadowsky gave a constructive proof for the existence of a developable Möbius band and posed the problem of determining the equilibrium configuration of a Möbius strip formed from an unstretchable material. He tackled this latter problem variationally and he deduced the bending energy for a strip whose width is much smaller than the length. This energy, now known as Sadowsky's energy, depends on the curvature and torsion of the centerline of the band and it is singular at the points where the curvature vanishes.

In this talk, we re-examine the derivation of the Sadowskys energy by means of the theory of Gamma-convergence. We obtain an energy that is never singular and agrees with the classical Sadowsky functional only for "large" curvature of the centerline of the strip.

The talk is based on ongoing joint work with L. Freddi, P. Hornung, and M.G. Mora.

Foundations of continuum mechanics

Friedemann Schuricht

Technische Universität Dresden, Germany

ABSTRACT

Peridynamics and Coleman & Noll's retardation theorem

Miroslav Šilhavý

Czech Academy of Science, Prague, Czech Republic

ABSTRACT

Peridynamics is a nonlocal continuum mechanics which replaces the differential operator embodied by the stress term $\operatorname{div}\mathbf{S}$ in Cauchy's equation of motion by a nonlocal force functional $\mathbf{L}$ to take into account long–range forces. The resulting equation of motion reads

$$\rho\ddot{\mathbf{u}} = \mathbf{L}\mathbf{u} + \mathbf{b}, \quad (\mathbf{u} = \text{displacement}, \mathbf{b} = \text{body force}, \rho = \text{density}).$$

It is conceivable that if the characteristic length δ of the interparticle interaction approaches 0, the theory reduces to the classical Cauchy's elasticity with $\mathbf{L}$ approaching $\operatorname{div}\mathbf{S}$. Moreover, one is interested in the asymptotic expansion of $\mathbf{L}$ in the powers of δ containing the higher gradients of $\mathbf{u}$ as the coefficients. Several results of this type exist for linear isotropic peridynamic materials, using the original integral form of $\mathbf{L}$ due to S. A. Silling and co–authors.

The paper proposes to enlarge the scope of the peridynamic theory by replacing Silling's form of $\mathbf{L}$ by a general, "unspecified" nonlocal, nonlinear functional of the displacement field $\mathbf{u}$ over the body. On the heuristic level this corresponds to passing from the binary interparticle forces to more general interactions involving also the ternary and generally multiparticle interactions. The limit of the characteristic length δ of forces approaching 0 is studied within this framework, based, like the preceding works, on the Taylor expansion of $\mathbf{u}$ at the given point of the continuum. However, the general form of $\mathbf{L}$ requires a more general approach. Such a framework exists and is embodied in the spatial version, due to B. D. Coleman, of the retardation theorem of Coleman & Noll. The latter was designed to discuss a specific question in the theory of viscoelasticity. The spatial version leads to the desired asymptotic expansion of $\mathbf{L}$ in a nonlinear series in the higher gradients of $\mathbf{u}$. The merits and demerits of this approach will be discussed. The merits are obvious: generality, nonlinearity and the inclusion of non–isotropic materials. Among the demerits is, for example, a group of negative mathematical results indicating that one should not be too cavalier in imposing freely the differentiability of nonlocal functionals. The other is that the expansion of the nonlocal energy need not be the energy for the expansion of the force.