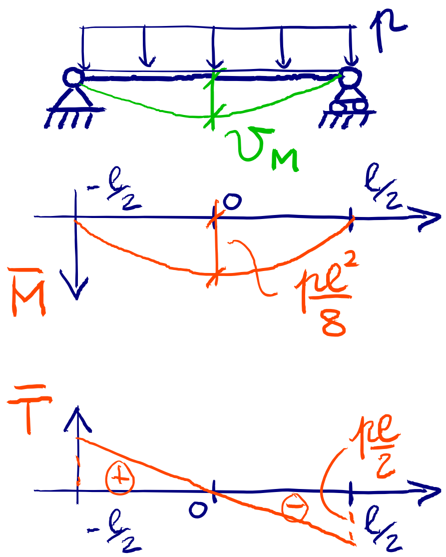
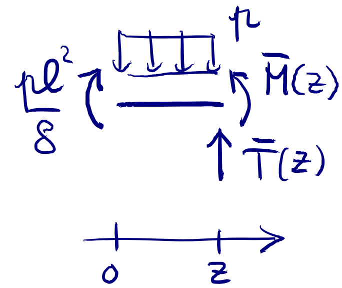
Esercizio: calcolare v_M, l'abbassamento in mezzeria

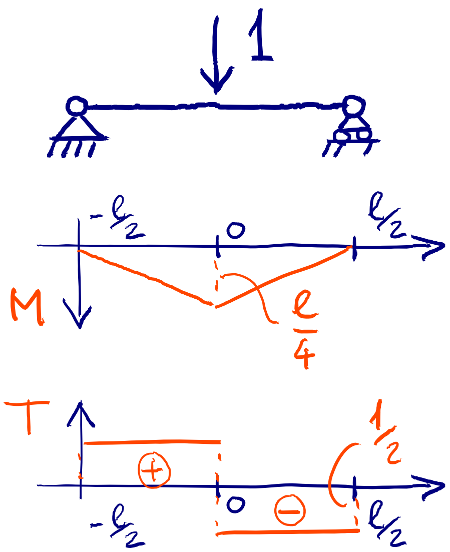


sistema congruente

$$\bar{M}(z) = \frac{p\ell^2}{8} - \frac{pz^2}{2} = \frac{p}{2}\left(\frac{\ell^2}{4} - z^2\right)$$



$$\bar{T}(z) = -pz$$



sistema equilibrato

$$M(z) = \frac{\ell}{4} - \frac{z}{2} \quad , \ z \in \left(0, \frac{\ell}{2}\right)$$

$$T(z) = -\frac{1}{2} \quad , \ z \in \left(0, \frac{\ell}{2}\right)$$

$$L_{v_{est}} = 1 \cdot v_M = L_{v_{int}} = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \left(\frac{T\bar{T}}{r_S} + \frac{M\bar{M}}{r_F}\right) dz =$$

$$= 2\int_0^{\frac{\ell}{2}} \left(\frac{T\bar{T}}{r_S} + \frac{M\bar{M}}{r_F}\right) dz = 2\int_0^{\ell/2} \left(\frac{pz}{2r_S} + \frac{p}{2r_F}\left(\frac{\ell^2}{4} - z^2\right)\left(\frac{\ell}{4} - \frac{z}{2}\right)\right) dz =$$

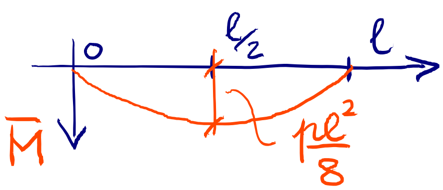
per la simmetria, il contributo sul tratto $\left(-\frac{\ell}{2}, 0\right)$ è uguale a quello sul tratto $\left(0, \frac{\ell}{2}\right)$

$$= \frac{p l^2}{8 r_s} + \frac{p}{r_F}\int_0^{l/2}\left(\frac{l^3}{16} - \frac{l^2 z}{8} - \frac{l z^2}{4} + \frac{z^3}{2}\right)dz =$$

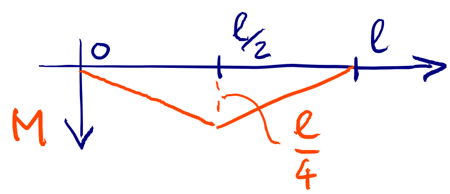
$$= \frac{p l^2}{8 r_s} + \frac{p}{r_F}\left[\frac{l^3 z}{16} - \frac{l^2 z^2}{8\cdot 2} - \frac{l z^3}{4\cdot 3} + \frac{z^4}{2\cdot 4}\right]_0^{l/2} =$$

$$= \frac{p l^2}{8 r_s} + \frac{p l^4}{r_F}\underbrace{\left(\frac{1}{16\cdot 2} - \frac{1}{8\cdot 2\cdot 4} - \frac{1}{4\cdot 3\cdot 8} + \frac{1}{2\cdot 4\cdot 16}\right)}_{= \frac{1}{32}\left(1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right) = \frac{1}{32\cdot 12}(12 - 6 - 4 + 3)} =$$

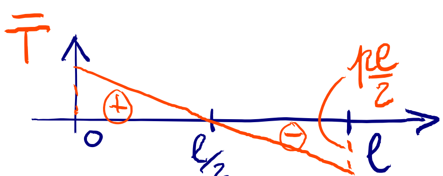
$$= \frac{p l^2}{8 r_s} + \frac{5}{384}\frac{p l^4}{EJ}$$



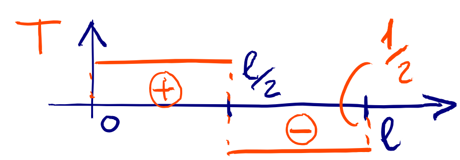
$\bar{M}(z) = \frac{p}{2}(lz - z^2)$



$M(z) = z/2,$
$z \in (0, \frac{l}{2})$



$\bar{T}(z) = p\left(\frac{l}{2} - z\right)$



$T(z) = \frac{1}{2},$
$z \in (0, \frac{l}{2})$

$$L_{Vint} = \int_0^l \left(\frac{T\bar{T}}{r_s} + \frac{M\bar{M}}{r_F}\right)dz = 2\int_0^{l/2}\left(\frac{T\bar{T}}{r_s} + \frac{M\bar{M}}{r_F}\right)dz =$$

come prima

$$= 2\left(\frac{p}{2r_s}\left[\frac{lz}{2} - \frac{z^2}{2}\right]_0^{l/2} + \frac{p}{4r_F}\left[\frac{lz^3}{3} - \frac{z^4}{4}\right]_0^{l/2}\right) =$$

$$= \frac{p l^2}{8 r_s} + \frac{p l^4}{2 r_F}\underbrace{\left(\frac{1}{3\cdot 8} - \frac{1}{4\cdot 16}\right)}_{= \frac{8-3}{3\cdot 64}} = \frac{p l^2}{8 r_s} + \frac{5}{384}\frac{p l^4}{r_F}$$