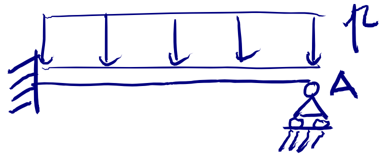
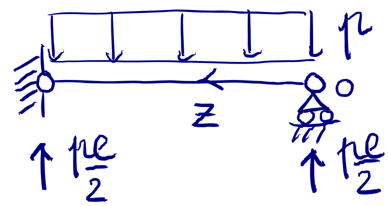
Sdc 15-11-13

esempio precedente

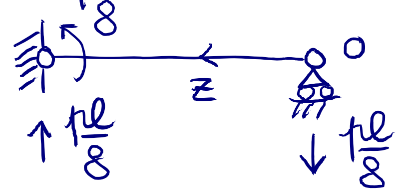


soluzione



+

$\frac{p\ell^2}{8} = X$



$$M(z) = M_0 + X M_1$$

$$M_0(z) = \frac{p}{2}(z^2 - \ell z) \qquad M_1(z) = \frac{z}{\ell}$$

calcolo di φ_A

sistema equilibrato

$$M_e(z) = -1$$

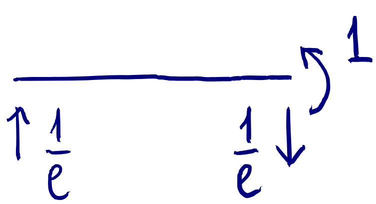
$$L_{V_{est}} = 1 \cdot \varphi_A = L_{V_{int}} = \int_{\mathcal{L}} M_e \frac{M_0 + X M_1}{r_F} = \qquad (\chi = 0)$$

$$= \int_0^{\ell} \left(\frac{p}{2r_F}(-z^2 + \ell z) + \frac{p\ell^2}{8r_F}\left(-\frac{z}{\ell}\right) \right) dz = \frac{p\ell^3}{r_F}\left(\frac{1}{2}\left(-\frac{1}{3} + \frac{1}{2}\right) - \frac{1}{8}\,\frac{1}{2} \right) =$$

$$= \frac{p\ell^3}{48 r_F}$$

$$\frac{1}{12} - \frac{1}{16} = \frac{4-3}{48} = \frac{1}{48}$$

sistema equilibrato alternativo



$$M_e(z) = -1 + \frac{z}{\ell}$$

$$\varphi_A = \int_0^{\ell} \left(\frac{p}{2r_F}(z^2 - \ell z)\left(-1 + \frac{z}{\ell}\right) + \frac{p\ell^2}{8r_F}\,\frac{z}{\ell}\left(-1 + \frac{z}{\ell}\right) \right) dz =$$

$$= \int_0^{\ell} \left(\frac{p}{2r_F}\left(-z^2 + \ell z + \frac{z^3}{\ell} - z^2\right) + \frac{p\ell^2}{8r_F}\left(-\frac{z}{\ell} + \frac{z^2}{\ell}\right) \right) dz =$$

$$= \frac{p\ell^3}{r_F}\left(\frac{1}{2}\left(-\frac{1}{3} + \frac{1}{2} + \frac{1}{4} - \frac{1}{3}\right) + \frac{1}{8}\left(-\frac{1}{2} + \frac{1}{3}\right) \right) = \frac{p\ell^3}{48 r_F}$$

$$\frac{1}{48}(-8 + 12 + 6 - 8 - 3 + 2) = \frac{1}{48}$$

Esempio: cedimento del vincolo

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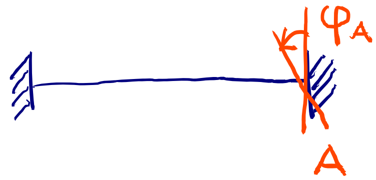
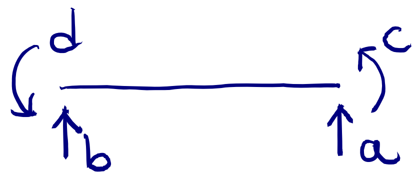


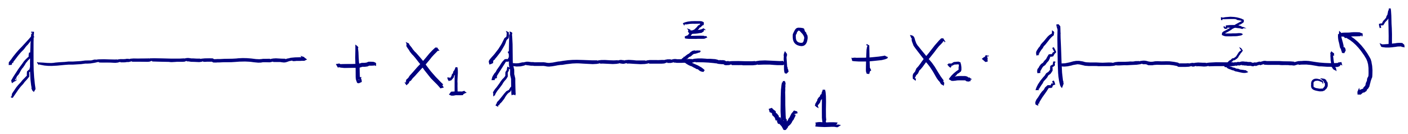


$a + b = 0$

$a\ell + c + d = 0$

∞^2 soluzioni

scegliendo $X_1 = a$, $X_2 = c$



$M_0 \equiv 0$ $\qquad M_1 = z$ $\qquad M_2 = -1$

$$\eta_{10} = 0 = \eta_{20} \qquad \eta_{11} = \int_0^\ell \frac{M_1^2}{r_F} dz = \frac{\ell^3}{3r_F} \qquad \eta_{22} = \int_0^\ell \frac{M_2^2}{r_F} dz = \frac{\ell}{r_F}$$

$$\eta_{12} = \int_0^\ell \frac{M_1 M_2}{r_F} dz = -\frac{\ell^2}{2r_F} = \eta_{21}$$

$$\eta_1 = 0 \quad , \quad \eta_2 = \varphi_A$$

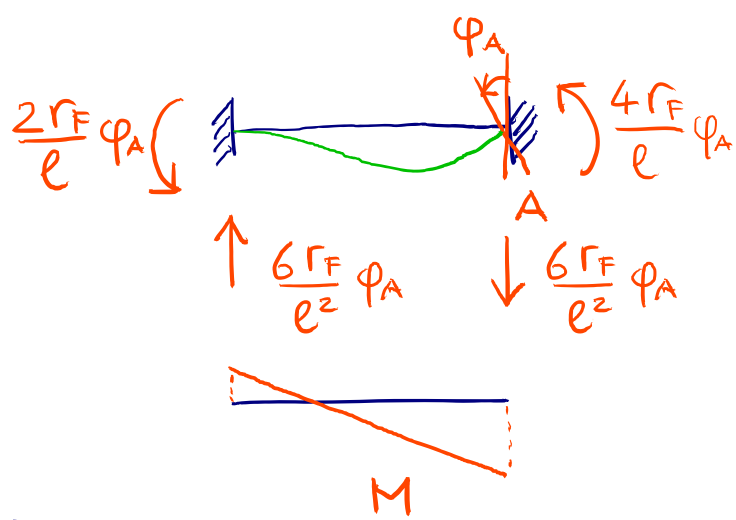
$$\begin{cases} X_1 \eta_{11} + X_2 \eta_{12} = 0 \\ X_1 \eta_{21} + X_2 \eta_{22} = \varphi_A \end{cases}$$

dalla prima equazione

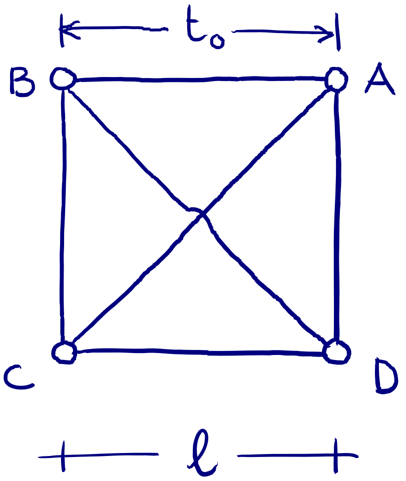
$$X_1 = -X_2 \frac{\eta_{12}}{\eta_{11}} = \frac{3}{2} \frac{X_2}{\ell}$$

$$X_2 \left(\frac{3}{2} \frac{1}{\ell} \left(-\frac{\ell^2}{2r_F} \right) + \frac{\ell}{r_F} \right) = \varphi_A$$

$$X_2 = \frac{4r_F}{\ell} \varphi_A \quad , \quad X_1 = \frac{6r_F}{\ell^2} \varphi_A$$

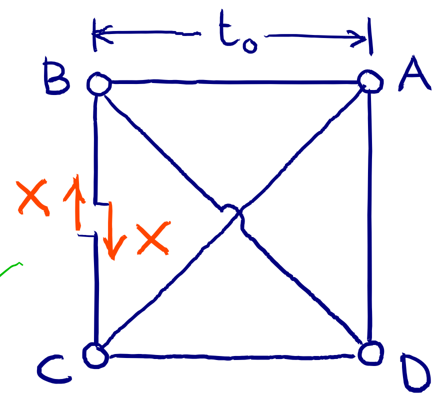


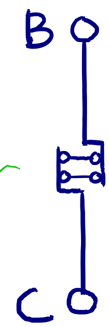
stessa r_E per ogni asta



$X = N_{BC}$

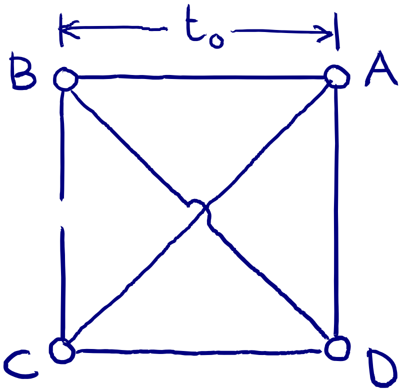
η_1 è lo spost. relativo tra le due sezioni





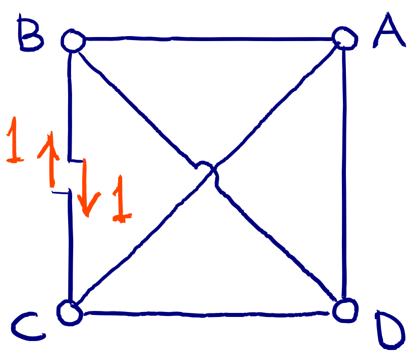
solo $N \neq 0$ taglio e momento sono nulli

sistema "0"



tutti gli sforzi normali sono nulli (deformazioni termiche, distorsioni e cedimenti non causano sollecitazioni in sistemi staticamente determinati, le equazioni di equilibrio hanno solo la soluzione nulla)

sistema "1"



risolvendo le equazioni di equilibrio:

$$N_{AB} = N_{AD} = N_{BC} = N_{CD} = 1$$

$$N_{AC} = N_{BD} = -\sqrt{2}$$

$$\eta_{10} = \sum_{a=1}^{N} \left(N_1 \varepsilon_o \ell\right)_a = \sum_{a=1}^{N} \left(N_1\left(\frac{N_o}{r_E} + \alpha t_o\right)\ell\right)_a = 1 \cdot \alpha t_o \ell$$

$$\eta_{11} = \sum_{a=1}^{N} \left(N_1 \varepsilon_o \ell\right)_a = \sum_{a=1}^{N} \left(\frac{N_1^2}{r_E}\ell\right)_a = 4 \cdot 1^2 \cdot \frac{\ell}{r_E} + 2\left(-\sqrt{2}\right)^2 \frac{\sqrt{2}\,\ell}{r_E} =$$

$$= 4\left(1+\sqrt{2}\right)\frac{\ell}{r_E}, \quad \eta_1 = 0 \implies X = -\frac{\eta_{10}}{\eta_{11}} = -\frac{\alpha t_o r_E}{4\left(1+\sqrt{2}\right)}$$