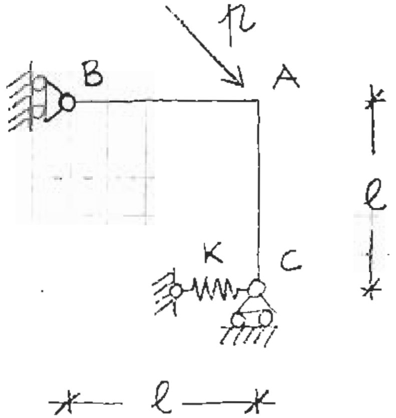
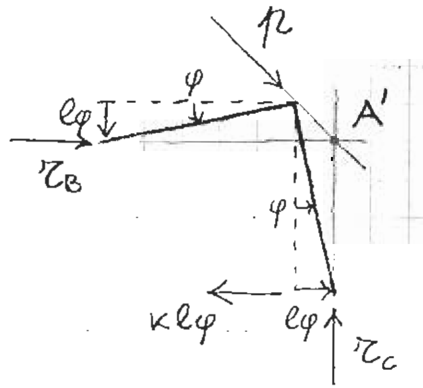
$\boxed{13}$





Equilibrio alla traslazione verticale:

$$r_C - P/\sqrt{2} = 0 \iff r_C = \frac{P}{\sqrt{2}}$$

Equilibrio alla traslazione orizzontale:

$$P/\sqrt{2} + r_B - k\ell\varphi = 0 \iff r_B = -\frac{P}{\sqrt{2}} + k\ell\varphi$$

Equilibrio alla rotazione intorno ad A:

$$r_B\,\ell\varphi + r_C\,\ell\varphi - k\ell^2\varphi = 0$$

$$\left(-\frac{P}{\sqrt{2}} + \cancel{k\ell\varphi} + \frac{P}{\sqrt{2}} - k\ell\right)\ell\varphi = 0 \iff k\ell^2\varphi = 0 \iff \varphi = 0$$

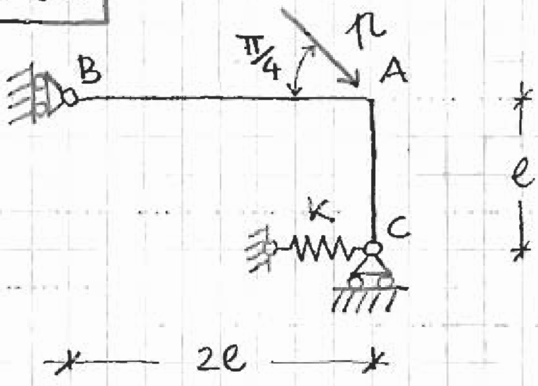
dá luogo ad un φ^2

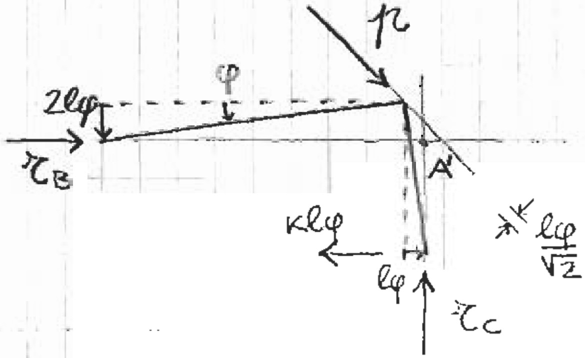
D'altra parte, scrivendo l'equilibrio alla rotazione intorno ad A':

$$k\ell^2\varphi\,(1-\cancel{\varphi}) = 0 \iff \varphi = 0$$

L'equilibrio è possibile solo nella configurazione indeformata.

13 bis





Come prima $r_C = \dfrac{p}{\sqrt{2}}$ e $r_B = -\dfrac{p}{\sqrt{2}} + K\ell\varphi$

Bilancio dei momenti intorno ad A:

$$r_B(2\ell\varphi) + r_C\,\ell\varphi - K\ell^2\varphi = 0$$

$$-\sqrt{2}\,p\,\ell\varphi + \cancel{2K\ell^2\varphi^2} + \frac{p}{\sqrt{2}}\ell\varphi - K\ell^2\varphi = 0$$

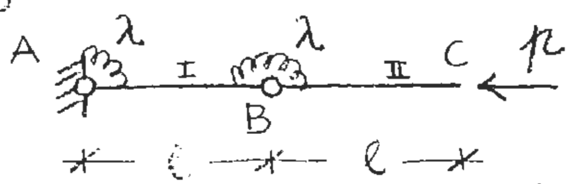
$$\Longleftrightarrow \quad \varphi\left(-\frac{1}{\sqrt{2}}\,p - K\ell\right) = 0 \quad ; \qquad p_c = -\sqrt{2}\,K\ell$$

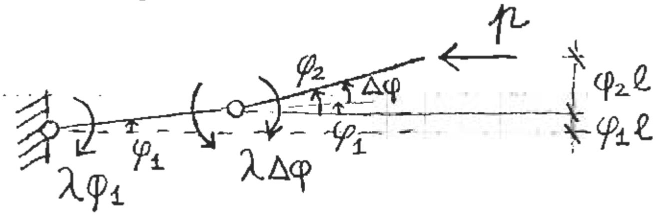
(Il carico che rende la struttura instabile è diretto in verso opposto a quello di figura)

Nella configurazione deformata, il punto A', intersezione delle rette d'azione di r_B e r_C, non appartiene alla retta d'azione di p (a differenza del caso precedente). Rispetto ad A', il momento dovuto a p deve essere bilanciato da quello dovuto alla reazione della molla:

$$-p\,\frac{\ell\varphi}{\sqrt{2}} - K\ell^2\varphi(1-\varphi) = 0 \quad \Longleftrightarrow \quad \varphi\left(p + \sqrt{2}\,K\ell\right) = 0$$

15





$$\Delta\varphi = \varphi_2 - \varphi_1$$

$$0 = M_A = p\ell(\varphi_1+\varphi_2) - \lambda\varphi_1$$

$$0 = M_B^{II} = p\ell\varphi_2 - \lambda(\varphi_2-\varphi_1)$$

$$\begin{bmatrix} p\ell - \lambda & p\ell \\ \lambda & p\ell - \lambda \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = 0$$

Affinché ci siano soluzioni $(\varphi_1, \varphi_2) \neq (0,0)$ il determinante della matrice deve essere nullo:

$$(p\ell-\lambda)^2 - p\ell\lambda = 0 \iff p^2 - 3\frac{\lambda}{\ell}p + \frac{\lambda^2}{\ell^2} = 0$$

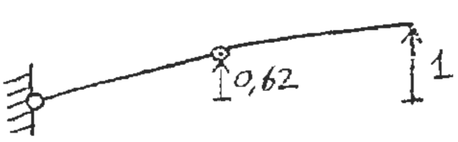
$$p_{1/2} = \frac{1}{2}\left(3\frac{\lambda}{\ell} \pm \frac{\lambda}{\ell}\sqrt{5}\right)$$

$$p_1 = \frac{3-\sqrt{5}}{2}\frac{\lambda}{\ell}, \quad p_2 = \frac{3+\sqrt{5}}{2}\frac{\lambda}{\ell} \quad ; \quad p_c = p_1 < p_2$$

Sostituendo $p = p_1$ nella 1ª equazione:

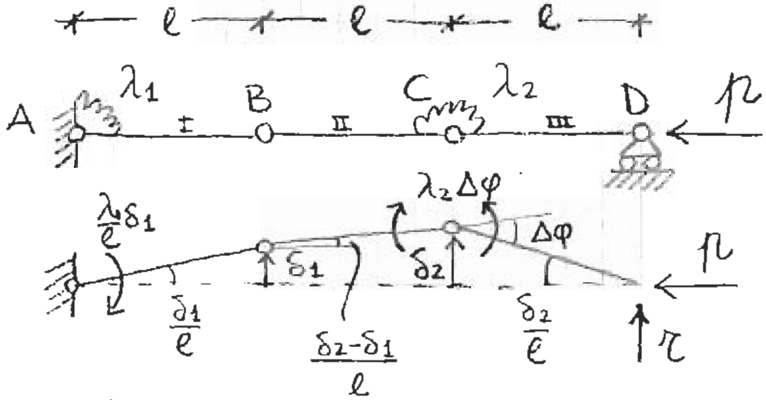
$$\left(\frac{3-\sqrt{5}}{2} - 1\right)\varphi_1 + \frac{3-\sqrt{5}}{2}\varphi_2 = 0$$

Ponendo $\varphi_2 = \varphi$, si ha $\varphi_1 = -\dfrac{3-\sqrt{5}}{1-\sqrt{5}}\varphi \simeq 0,62\,\varphi$



(autovettore associato a p_1)

17



$$\Delta\varphi = \frac{\delta_2}{l} + \frac{\delta_2-\delta_1}{l} = \frac{2\delta_2-\delta_1}{l}$$

$$M_A = \tau(3l) - \frac{\lambda_1}{l}\delta_1 = 0 \iff \tau = \frac{\lambda_1}{3l^2}\delta_1$$

$$M_B^{\mathrm{II\,III}} = -p\,\delta_1 + \tau(2l) = 0 \iff \delta_1\left(p - \frac{2}{3}\frac{\lambda_1}{l}\right) = 0$$

$$M_C^{\mathrm{III}} = -p\,\delta_2 + \tau l + \frac{\lambda_2}{l}(2\delta_2 - \delta_1) = 0$$

$$\iff -p\,\delta_2 + \frac{\lambda_1}{3l}\delta_1 + \frac{\lambda_2}{l}(2\delta_2 - \delta_1) = 0$$

$$\begin{bmatrix} p - \frac{2}{3}\frac{\lambda_1}{l} & 0 \\ -\frac{\lambda_1}{3l} + \frac{\lambda_2}{l} & p - 2\frac{\lambda_2}{l} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = 0$$

$$\left(p - \frac{2}{3}\frac{\lambda_1}{l}\right)\left(p - \frac{2\lambda_2}{l}\right) = 0 \quad ; \quad p_1 = \frac{2}{3}\frac{\lambda_1}{l}, \quad p_2 = \frac{2\lambda_2}{l}$$

$$p = p_1, \quad \cancel{\left(-\frac{\lambda_1}{3l} + \frac{\lambda_2}{l}\right)}\delta_1 + 2\cancel{\left(\frac{\lambda_1}{3l} - \frac{\lambda_2}{l}\right)}\delta_2 = 0 \iff \delta_2 = \frac{\delta_1}{2}$$

$$p = p_2, \quad \delta_1 = 0$$

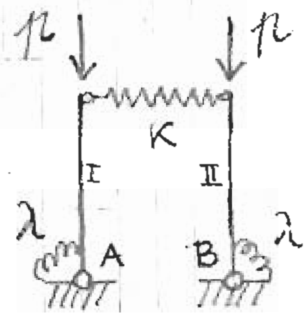
$p = p_1$

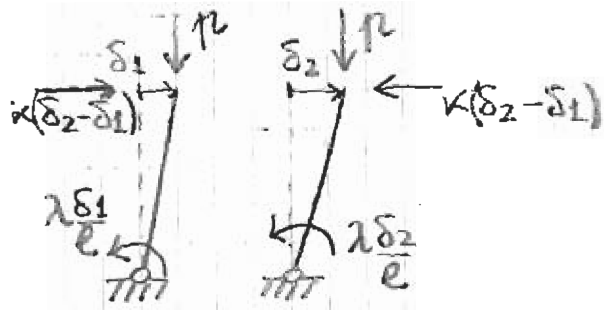
$p = p_2$

$$\frac{2}{3}\frac{\lambda_1}{l} \gtrless \frac{2\lambda_2}{l} \quad ; \quad \lambda_1 \gtrless 3\lambda_2$$

Se $\lambda_1 < 3\lambda_2$ allora $p_c = p_1$.

20





$$M_A^{I} = \lambda \frac{\delta_1}{\ell} - P\delta_1 - K\ell(\delta_2 - \delta_1) = 0$$

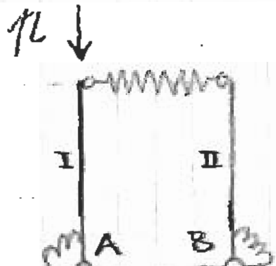
$$M_B^{II} = \lambda \frac{\delta_2}{\ell} - P\delta_2 + K\ell(\delta_2 - \delta_1) = 0$$

Sommando: $(\delta_1 + \delta_2)\left(\lambda/\ell - P\right) = 0$

Sottraendo: $(\delta_2 - \delta_1)\left(2K\ell - P + \frac{\lambda}{\ell}\right) = 0$

$$\left.\begin{aligned} P_1 &= \frac{\lambda}{\ell}, \quad \delta_2 = \delta_1 \\ P_2 &= \frac{\lambda}{\ell} + 2K\ell, \quad \delta_2 = -\delta_1 \end{aligned}\right\} \iff P_c = P_1$$

20 bis



$$M_B^{II} = \frac{\lambda}{\ell}\delta_2 + K\ell(\delta_2 - \delta_1) = 0$$

$$\iff \delta_1 = \left(1 + \frac{\lambda}{K\ell^2}\right)\delta_2$$

$$M_A^{I} = \left[\left(-P + \frac{\lambda}{\ell}\right)\left(1 + \frac{\lambda}{K\ell^2}\right) - K\ell\left(-\frac{\lambda}{K\ell^2}\right)\right]\delta_2 = 0$$

$$-P_c\left(1 + \frac{\lambda}{K\ell^2}\right) + \frac{2\lambda}{\ell} + \frac{\lambda^2}{K\ell^3} = 0$$

$$P_c = \frac{2 + \frac{\lambda}{K\ell^2}}{1 + \frac{\lambda}{K\ell^2}}\,\frac{\lambda}{\ell} = \frac{\lambda}{\ell}\left(\frac{1}{1 + \frac{\lambda}{K\ell^2}} + 1\right)$$

$K \to 0,\ P_c \to \frac{\lambda}{\ell}$; $K \to +\infty,\ P_c \to 2\frac{\lambda}{\ell}$