

# Modular Tensegrity Structures: The TorVergata Footbridge

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**Abstract.** Tensegrity systems are lightweight, deployable or adaptive structures. To improve their stiffness, modular tensegrity structures are assembled with the use of strut-to-strut joints and supplementary cables. These concepts are applied to designing the TorVergata Footbridge.

## 1 Introduction

As a structure class, *tensegrity systems* are spatial trusses composed by struts and cables, such that: (1) the collection of cables appears as a connected set (**tensile-integrity**); (2) the struts are never connected to each other (floating compression); (3) there are infinitesimal mechanisms, stabilized by self-stress states. Certain authors regard properties (1) and (2) as essential, others insist only on (1); others do not include anyone of the two into their definitions. It seems to us, however, that the importance of a feature strictly related to point (3) must be stressed, namely, the *form-finding property*:

given a  $n_e$ -element tensegrity system, if the lengths of  $(n_e - 1)$  elements are fixed, then a stable equilibrium configuration (a so-called *tensegrity configuration*) obtains when the last cable (strut) has minimal (maximal) length.

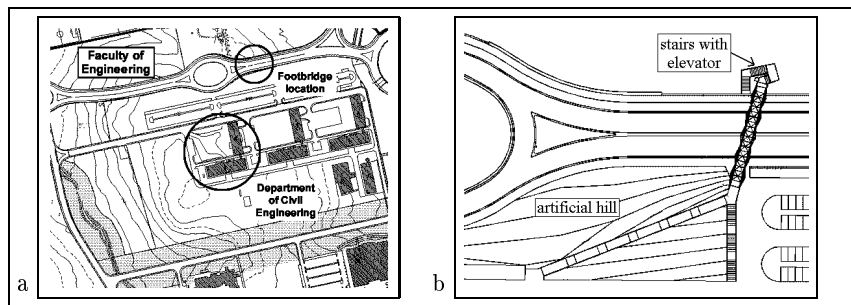
For a fixed topology (i.e., once a collection of joints connected by struts and cables is chosen), if there is more than one stable configuration, then the system passes from one configuration to another by changing simultaneously the length of two or more elements. This property opens the way to posing the Form-Finding Problem: to find the set of all possible tensegrity configurations for a system of fixed topology.

The form-finding property of tensegrity systems, with their related ability to change their shape, suggests to use tensegrity systems when it is desirable to have deployable structures (more generally, structures with a variable geometry), or smart structures some elements of which serve as sensors or actuators. However, these systems are complex and their stiffness is generally low. Complexity is due to: (i) the particular geometry; (ii) the special design required for the joints; (iii) the highly nonlinear mechanical behaviour; moreover, (iv) the form-finding problem is difficult to solve, except for cases of especially high symmetry. Low stiffness is due to: (v) the presence of infinitesimal mechanisms; (vi) the high ratio, with respect

to conventional trusses, between the number of joints and the number of elements; (vii) the absence of strut-to-strut connections.

Complexity and low stiffness make civil-engineering applications problematic. To design a tensegrity system fulfilling requirements usually mandatory for civil structures, we must make it simple and of adequate stiffness. We choose to relax our definition above, and consider *modular tensegrity structures*: as a whole, these structures are not tensegrity systems, but are composed of tensegrity modules, assembled with the use of strut-to-strut connections and supplementary cables.

This paper illustrates the design of the *TorVergata Footbridge*, a new, nontrivial example of a modular tensegrity structure. The TVF is to be built near the Faculty of Engineering of the University of TorVergata in Rome; it will allow pedestrians to cross safely one of the main traffic arteries of the Campus (Fig. 1).



**Fig. 1.** The location of the TorVergata Footbridge.  
a) Plan of the current state of the School of Engineering b) Detail of the final state.

A footbridge is a permanent facility that need not be realized by employing tensegrity concepts. Our choice has been influenced by the sense of transparency and lightness emanating from the large tensegrity sculp/structures realized by Kenneth Snelson, which seem observable not only from a distance but also from their inside: we have chosen a structure people can become familiar with as they walk through it.

To design any tensegrity structure - in particular, a tensegrity footbridge - is a highly unusual task, because there are no codified rules. Since the solutions of the static and dynamic design problems depend strongly on each other and on the prestress design, structure optimization can be achieved only by a trial-and-error process; moreover, the final solution depends on the choice of the module. For these reasons, we here refrain from stating rules of general validity, not even *a posteriori*. We limit ourselves to present our results in an order we find convenient.

## 2 Tensegrity characterization

In this section we briefly summarize some useful concepts about the algebraic characterization of tensegrity systems.

The main information on a pin-jointed system is contained in the *structural matrix*, that is, either the *equilibrium matrix* (representing the linear relation between the axial forces on the elements and the external loads acting on the joints) or its transpose, the *compatibility matrix* (the linear relation between displacements of joints and deformations of elements).

The *structural matrix of a tensegrity configuration has not full rank*; this *indeterminacy condition* implies that there exist at least a mechanism (the joint displacements causing null deformations of the elements) and a self-stress state (the axial forces balanced by null external loads).

The *tensegrity configuration is stable*, in that the *self-stress state geometrically stiffens every mechanisms*<sup>1</sup>.

The analysis of the structural matrix leads to the known *extended Maxwell's rule*, that is, the following relation between the number of joints ( $n_j$ ), elements ( $n_e$ ), independent mechanisms ( $n_m$ ) and self-stress states ( $n_s$ ):

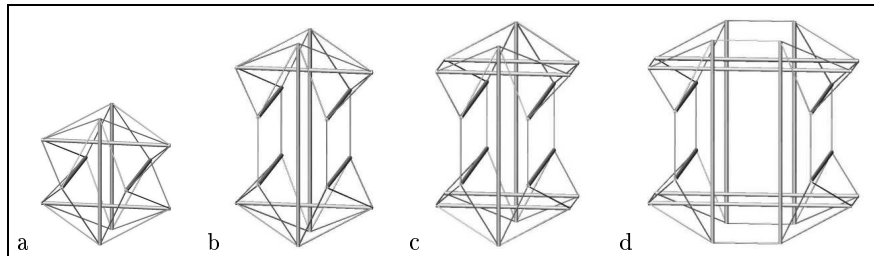
$$3n_j - 6 - n_e = n_m - n_s, \quad (1)$$

where 6 is the number of rigid body motions in space.

We note that the affine transformations of space (those preserving parallelism) do not affect the indeterminacy and stability conditions.

### 3 The re-expanded octahedron

The module chosen for the footbridge consists in a modified version of the so-called *expanded octahedron*. The expanded octahedron is composed of 6 struts and 24 cables and it is shown in Fig. 2a. The name was coined to reflect the fact that, if couples of parallel struts are joined together, then cables form the edges of an octahedron. The footbridge module (Fig. 2b) is obtained by a further 'expansion'.



**Fig. 2.** Expanded octahedra.

The module reduces to the previous one if the six vertical elements, four cables

<sup>1</sup> In the literature, this stability condition is stated in a number of ways, see for example [3].

and two struts, are so shortened that the four horizontal parallel struts come to be two-by-two superposed. Each of the modules in Figs. 2c and 2d reduces to the previous one by a similar mental way.

### 3.1 Form-finding solution

The extended Maxwell's rule computed for each of these modules gives for the difference  $(n_m - n_s)$  the following values: 0, 6, 12, 18. In the tensegrity configuration each of these system acquires a self-stress state; the number of mechanisms is respectively equal to 1, 7, 13, 19.

A method to satisfy the indeterminacy condition is to find the solutions of the equilibrium equations for null external loads. This shows that the only relevant parameters are  $\alpha_x$ ,  $\alpha_y$  and  $\alpha_z$ ; these are illustrated in Fig. 3 with reference to the last module shown in Fig. 2. For example,  $\alpha_y$  is given by the ratio between the following lengths: the distance between struts along the  $z$  axis and struts along the  $x$  axis; the distance between struts along the  $z$  axis and cables along the  $x$  axis. It can be shown that, for stable configurations, these ratios take values between 0 and 1. Then, the solution of the form-finding problem is:

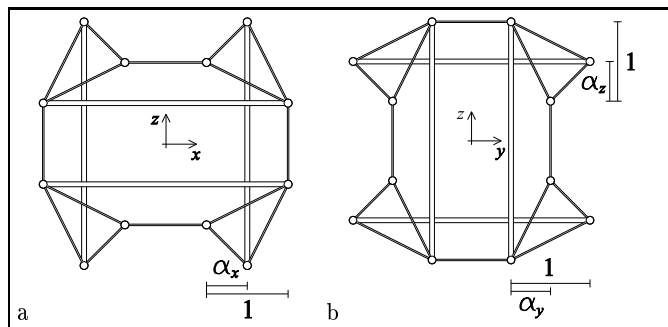


Fig. 3. Module parameters.

$$\alpha_x \alpha_y \alpha_z = (1 - \alpha_x)(1 - \alpha_y)(1 - \alpha_z) . \quad (2)$$

We note that during the aforementioned expansion, each intermediate configuration is still a tensegrity configuration. In fact, the important property of this *re-expanded* module is that the solution of the form-finding problem does not depend on the shortening length of the six vertical elements. This means that each of the three re-expanded modules possesses an empty space in its interior whose dimensions can be arbitrarily fixed. This fact make these modules well suited to contain both a deck and the users of the footbridge.

### 3.2 Assembling modules

The stiffer module is chosen, i.e., the module with the minimum number of mechanisms (Fig. 2b). The orientation of the module is such that the four parallel struts are vertical (Fig. 4). The opening in the module is fixed to be 3 meters wide and 2.6 meters high, enough to accommodate the deck and to be comfortable for the people (Fig. 4a).

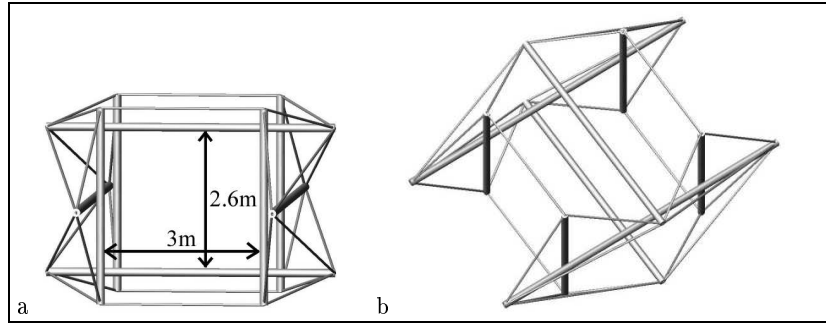


Fig. 4. Two views of the footbridge module.

These module are simple because their struts are mutually orthogonal. Assembling modules by pin-jointing two aligned struts permits to obtain some straight ‘compressive paths’ in the resulting structure. Some supplementary cables are employed to realize straight ‘tensile paths’ (Fig. 5a).

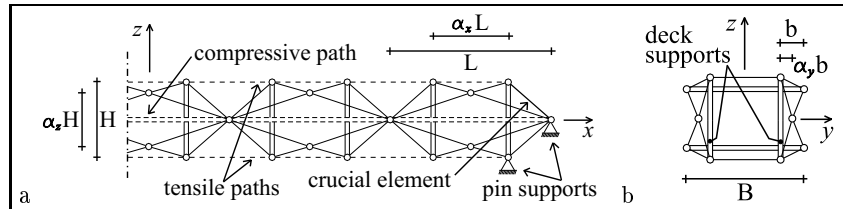
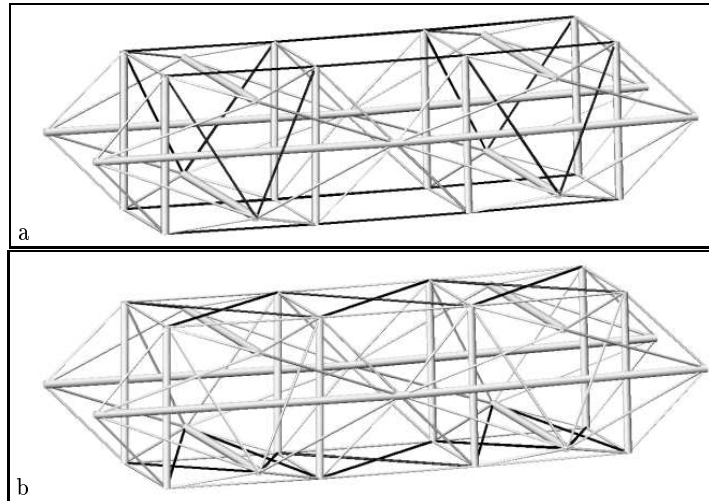


Fig. 5. (a) Side view of part of the structure showing modules assembled with some supplementary elements. (b) Transverse view of the structure.

The purpose of the supplementary cables is to eliminate all the mechanisms and to make the structure stiff enough to resist external actions. Figure 6 shows the positions of all supplementary cables.



**Fig. 6.** Assembling two modules with supplementary elements (depicted in black). Step (a), tensile elements and V-cables are added to stiffen the structure against vertical actions. Step (b), cross elements are added to stiffen the structure against horizontal and torsional actions.

## 4 Design of the modular structure

The design aims to maximize the overall stiffness of the footbridge. A simple way to estimate it is to measure the maximum displacement under vertical dead plus live loads. All static computation are carried out by taking into account the geometric stiffness due to the prestress, working with a large-displacement elastic analysis program written in MATLAB.

The whole structure spans a distance of  $L_{tot} = 32m$  without intermediate supports. The external supports consists in eight pins, four at each end, located as depicted in Fig. 5a.

Five equal modules compose the footbridge. This number realizes a compromise between the higher stiffness of structure with less modules and the higher cost of one with more.

### 4.1 Form-finding parameters

The best way to transfer the vertical loads to the structure is to place the supports of the deck on the four vertical struts, avoiding bending moments (Fig. 5b). If the  $x$  axis is the longitudinal axis of the footbridge, then the ratio  $\alpha_x$  is fixed equal to 0.5, so that both the vertical struts and the supports of the deck are equally spaced in the longitudinal direction: the footbridge looks regular. Considering the placement of the external supports, the longitudinal dimension of a module is  $L = 7.1m$ .

With this choice of  $\alpha_x$ , the form-finding condition (2) simplifies to:

$$\alpha_y + \alpha_z = 1, \quad (3)$$

where the  $z$  axis is vertical.

The choice of  $\alpha_z$  aims to minimize the height-to-span ratio ( $H/L_{tot}$ ) of the footbridge, in order to have a slender structure. Increasing the values of  $\alpha_z$  corresponds to decreasing the values of  $H = \alpha_z^{-1} 2.6m$ ; the stiffness of the footbridge decreases as well. Thus, the maximum allowable value of  $\alpha_z$  is 0.76; the height results to be  $H = 3.6m$ . The value of  $\alpha_z$  leads, by (3), to  $\alpha_y = 0.24$ .

Having fixed  $\alpha_y$ , a smaller transverse dimension  $b$  (see Fig. 5a) of the footbridge corresponds to higher stiffness. This value cannot be smaller than  $1.15m$ , to leave a visible clearance between struts along the  $x$  axis and those along the  $z$  axis.

## 4.2 Banana shape

The main axis of the footbridge has a *banana shape* (see Fig. 7). The maximum slope at the end modules is 5% (people can easily clear that). A comparison with the case of a straight axis reveals that this solution increases the geometric stiffness and reduces by 40% the maximum displacement. The negative counterpart is that the horizontal reaction at supports is increased by a similar percentage.

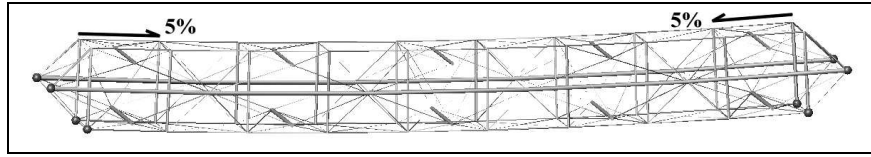


Fig. 7. Side view of the footbridge structure.

## 4.3 Types of element

The tensile elements composing the structure are of two kind: cables and traction bars. Because of the high breaking load of cables, traction bars with the same ultimate tensile load have a bigger area and hence an higher stiffness. Two traction bars of  $72\text{ mm}$  in diameter are found in each end module (Fig. 5a); these elements are crucial to the stiffness and strength of the structure. Traction bars of  $50\text{ mm}$  in diameter are employed as elements composing the tensile paths; traction bars of smaller diameter are employed as cross elements under the deck (Fig. 6b). The remaining tensile elements are cables of  $12\text{ mm}$  and  $24\text{ mm}$  in diameter. The struts have a circular hollow section whose diameter ranges from  $114\text{ mm}$  to  $193\text{ mm}$  and the corresponding wall thickness ranges from  $8\text{ mm}$  to  $12\text{ mm}$ .

## 5 Results of the analysis

To compare this structural solution to the traditional solutions, we compute the footbridge “specific weight”, that is, the ratio between the total weight of the modular structure (not considering the deck) and the total area of the deck surface. This ratio turns out to be around  $105 \text{ kg/m}^2$ , a value close to those of standard footbridge construction systems.

In this section we report the results of prestress, static and dynamic designs.

### 5.1 Applied prestress

The prestress allows the cables to take compressive (unloading) actions and increases the geometric stiffness of the structure. It may, however, cause instability problems in the struts. The prestress is applied to the structure in two steps. Firstly, to each individual module, after it is assembled; secondly, to the whole structure, by tensioning the supplementary cables.

Since each module possess only one self-stress state, all the elements become prestressed when the last element is tensioned. Then it is convenient to tension the cable with the lower tensile force. The prestress applied to each module corresponds to  $650 \text{ kN}$  of maximum compressive force in longitudinal struts (along  $x$  axis), while the tensile forces in the cables vary between  $140 \text{ kN}$  and  $280 \text{ kN}$ . The prestress applied to the whole structure is lower, because of the positive effect of the banana shape against vertical loads. The applied prestress is about  $65 \text{ kN}$  in the cross-cables placed over the deck and  $130 \text{ kN}$  in the V-cables (Fig. 6).

### 5.2 Statics

The most severe static load condition consists of the vertical dead and live loads plus the 60% of the horizontal wind load; the maximum vertical displacement computed is less than  $8 \text{ cm}$ . Other load conditions examined are: full wind load, seismic load, non-symmetric crowding.

A maximum axial force of  $760 \text{ kN}$  is carried by the struts; a maximum tensile force of  $850 \text{ kN}$  is reached in the traction bars. The resultant of the horizontal reactions at supports is  $1200 \text{ kN}$ .

### 5.3 Dynamics

A linearized analysis has been performed to compute the modal frequencies of the footbridge; these are shown in Table 1 together with the corresponding modal shapes. Such frequencies are sufficiently higher than those of the excitation loads due to wind and people.

The wind acts on the footbridge horizontally; its frequency is approximately  $1 \text{ Hz}$ . It is then reasonable to disregard its action, because the first horizontal modal frequency is  $3.65 \text{ Hz}$ . The problem often encountered with recently built lightweight footbridges is the synchronization problem of the step frequency of walking persons with the modal frequencies of the structure. A walking person loads the footbridge both horizontally and vertically, with frequencies that vary about  $1 \text{ Hz}$  and  $2 \text{ Hz}$

Mode	Frequency	Modal shape
1	3.65	horizontal
2	3.82	vertical
3	4.36	horizontal
4	6.51	torsional
5	6.63	horizontal 1 node
6	7.39	vertical 1 node
7	7.43	horizontal 1 node

**Table 1.** Modal frequencies and shapes of the footbridge.

respectively. These values have to be compared with the corresponding values for the footbridge. In our case, it could be possible to have resonance phenomena caused by the second harmonic of the vertical loading action; this harmonic is about  $4\text{ Hz}$ , near to the first vertical modal frequency. Simple computations have been carried out for two resonance hypothesis: (1) fifty people walking synchronously at midspan; (2) three people jumping simultaneously at midspan. In both cases, for a structural damping ratio of 0.003, the maximum vertical displacement was found to be about  $10\text{ cm}$ . Although this analysis is far from being exhaustive, it does give a first estimation of dynamic effects.

## 6 Construction process

Figures 8a and 8b are computer views of the final arrangement of the footbridge. At this preliminary stage of the project, the construction process is intended to be divided into three steps.

1. Each module is assembled in the plant. For ease of transportation, some elements are detached from the module allowing for folding into a compact bundle. Those elements are equipped with a length adjustment system, in order to realize the desired shape and calibrate the prestress of the module.
2. On location, the footbridge assembling is completed on the ground. The modules are assembled together and supplementary elements are added. The deck is installed in the interior of the modular structure.
3. The whole system is lifted so as to place on the supports. The supporting joints are equipped with a sliding regulation system for precise installation, before blocking them to their final positions.

With a view toward speeding up the construction process, all-at-once deploying strategies for the whole system are currently under study.

## Acknowledgements

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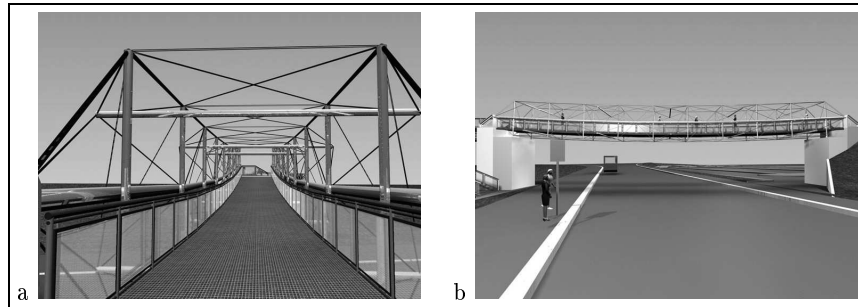


Fig. 8. .

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